Aljabar Vektor

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Institut Teknologi Sepuluh Nopember

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Overview

Besaran vektor

Dua vektor sama

Jumlah dua vektor: $\vec{a} + \vec{b}$

Selisih dua vektor: $\vec{a} - \vec{b}$

Vektor satuan

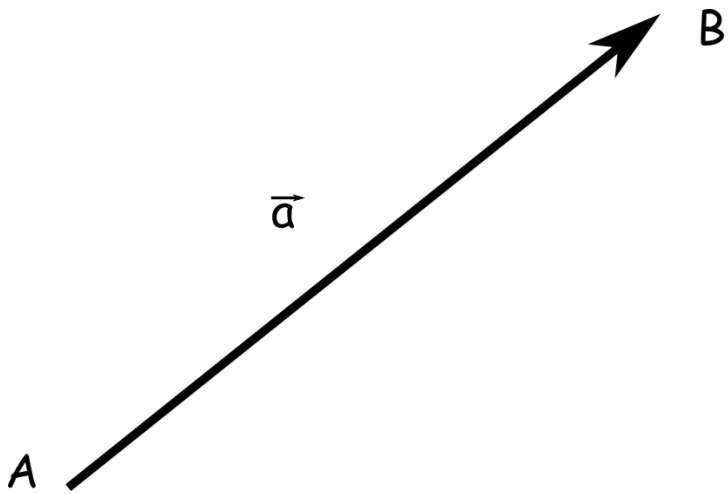
Perkalian titik (Dot product)

Perkalian silang (Cross product)

Besaran Vektor

Besaran

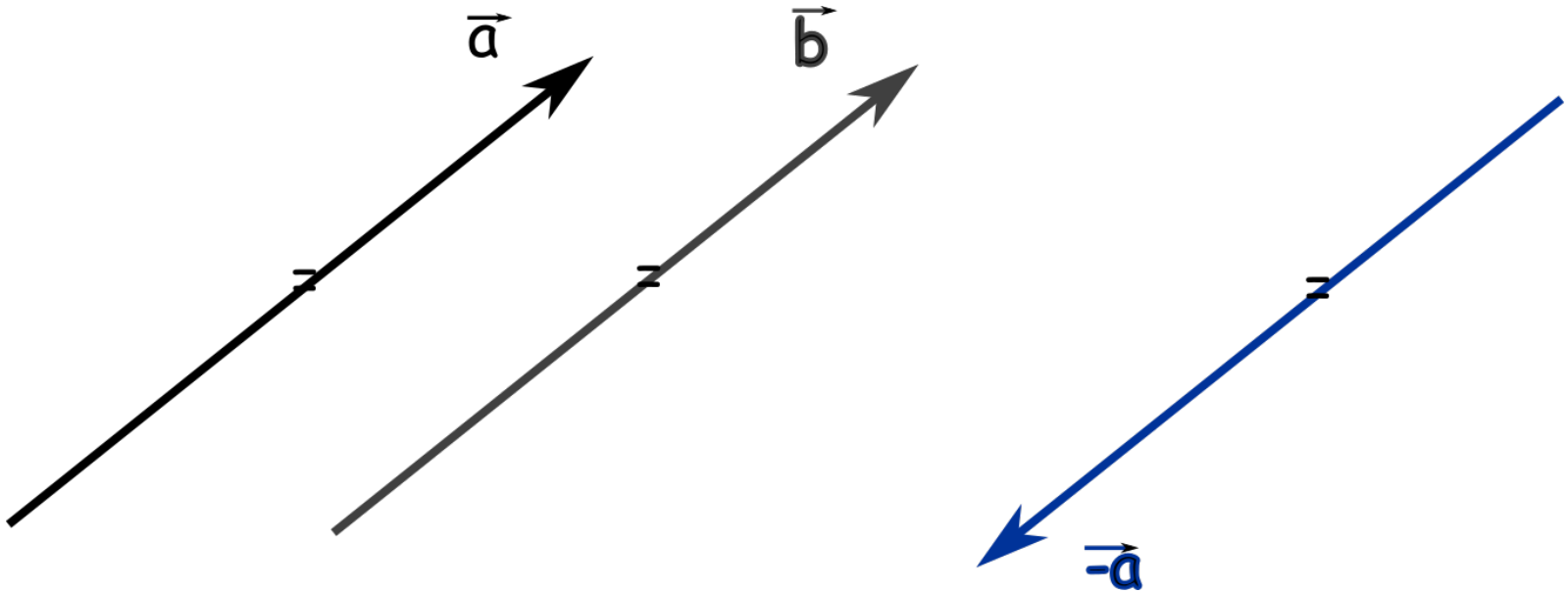
- Vektor: Kuantiti yang memiliki besar dan arah
  Contoh: kecepatan, percepatan, gaya.
- Skalar: Kuantiti yang hanya memiliki besar saja.
  Contoh: waktu, temperatur, massa, panjang.



$\overrightarrow{AB} = \vec{a}$; $|\vec{a}| = a =$ panjang vektor $\vec{a}$

Dua vektor sama

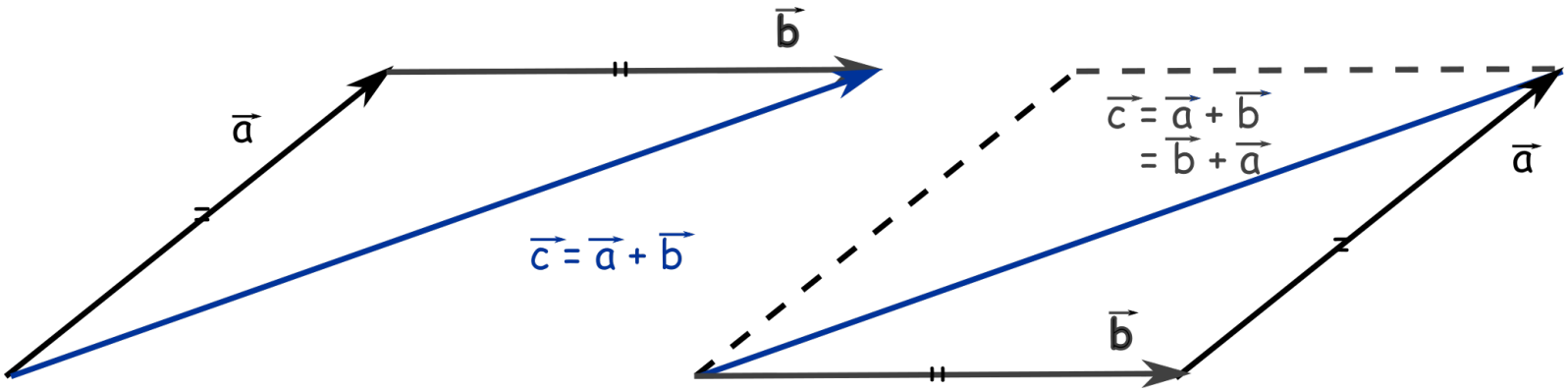
$\vec{a} = \vec{b} \Longrightarrow$ searah, sama panjang



$\overrightarrow{-a}$, sama panjang tetapi berlawanan arah dengan $\vec{a}$

Jumlah dua vektor: $\vec{a} + \vec{b}$

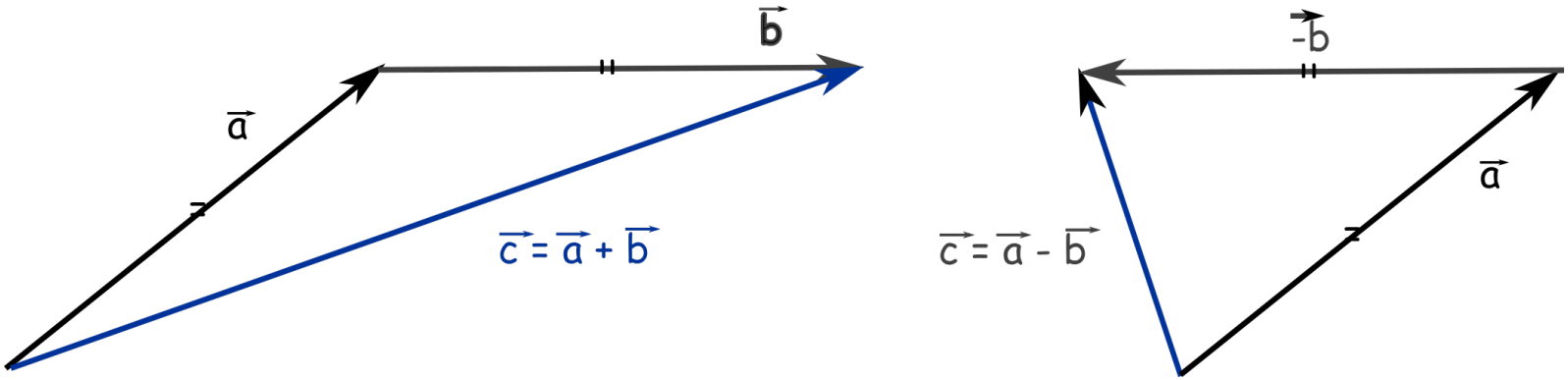
$\longrightarrow \vec{b}$ dimulai dari ujung vektor $\vec{a}$, vektor $\vec{b} \# \vec{a}$, lalu hubungkan pangkal $\vec{a}$ dengan ujung $\vec{b}$ tsb.



$$\boxed{\vec{a} + \vec{b} = \vec{b} + \vec{a}}$$

Selisih dua vektor: $\vec{a} - \vec{b}$

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = \vec{a} \quad \textit{ditambah} \quad -\vec{b}$$

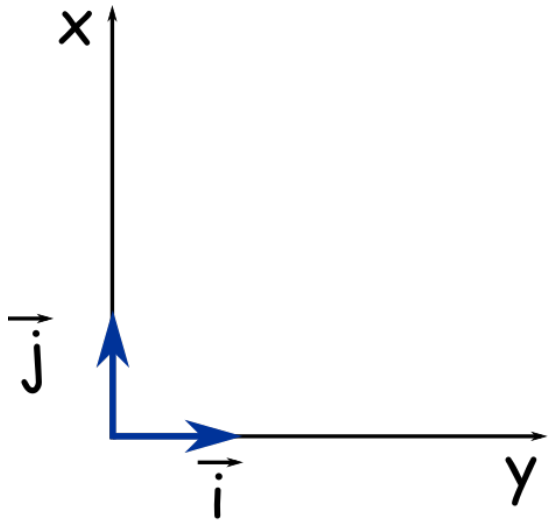


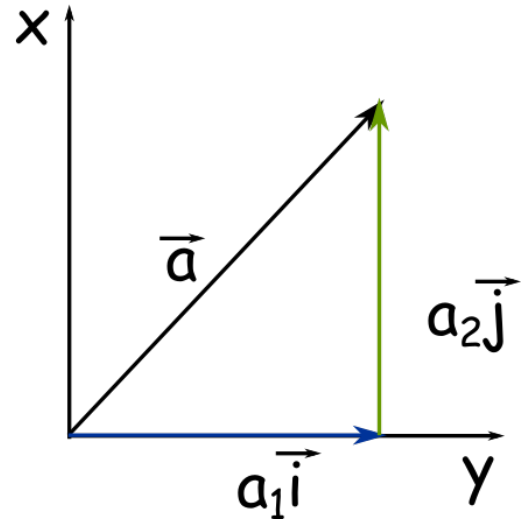
Jika $\vec{a} = \vec{b} \Longrightarrow \vec{a} - \vec{b} = \vec{0}$

Vektor nol, panjang nol, arah tak didefinisikan

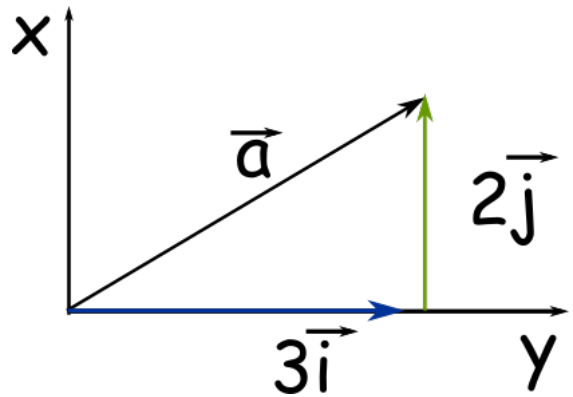
Vektor Satuan

Vektor satuan $\vec{i}$: Vektor dari titik (0,0) sampai titik (1,0).
Vektor satuan $\vec{j}$: Vektor dari titik (0,0) sampai titik (0,1).



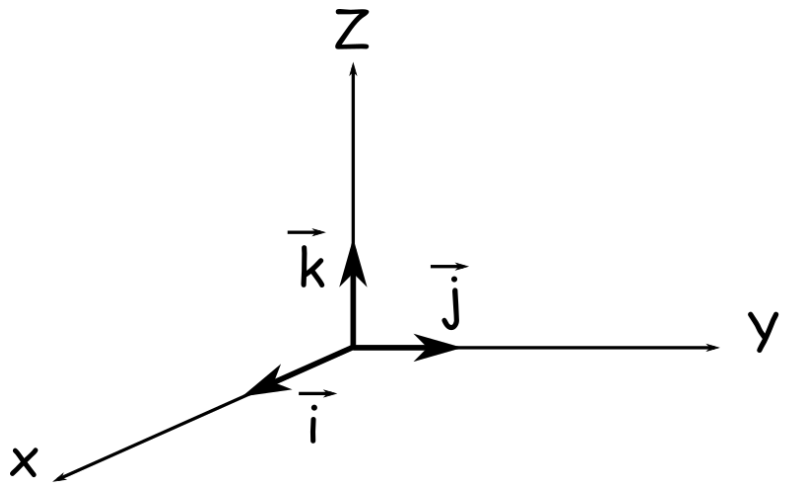


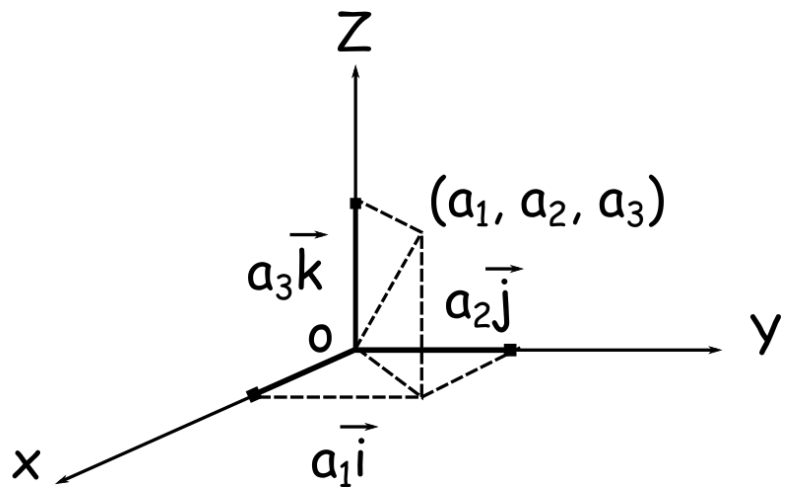
$\vec{a} = a_1\vec{i} + a_2\vec{j}$, Contoh: $\vec{a} = 3i + 2j$



Unit vektor siku-siku

$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$





$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

Vektor posisi $\vec{r}$ dari O ke $P(x, y, z)$ adalah:
$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

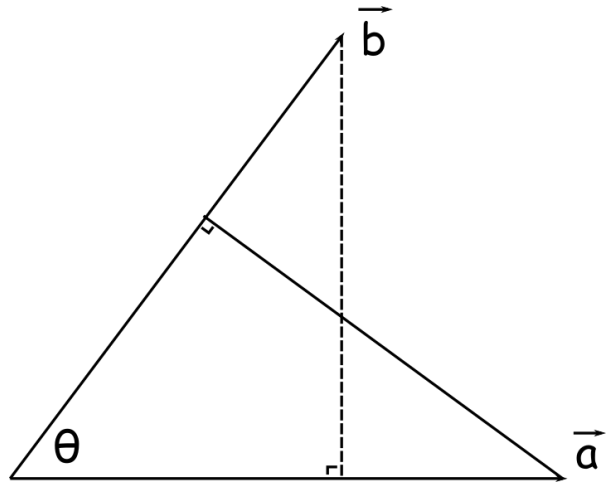
dengan panjang $r = \sqrt{x^2 + y^2 + z^2}$; $a = |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Perkalian titik (Dot product)

Definisi: $\boxed{\vec{a}.\vec{b} = ab\cos\theta}$; $(0 \leq \theta \leq \pi)$

$\vec{a}.\vec{b} = \vec{b}.\vec{a} \Longrightarrow \vec{i}.\vec{i} = \vec{j}.\vec{j} = \vec{k}.\vec{k} = 1$

$\vec{i}.\vec{j} = \vec{j}.\vec{k} = \vec{k}.\vec{i} = 0$

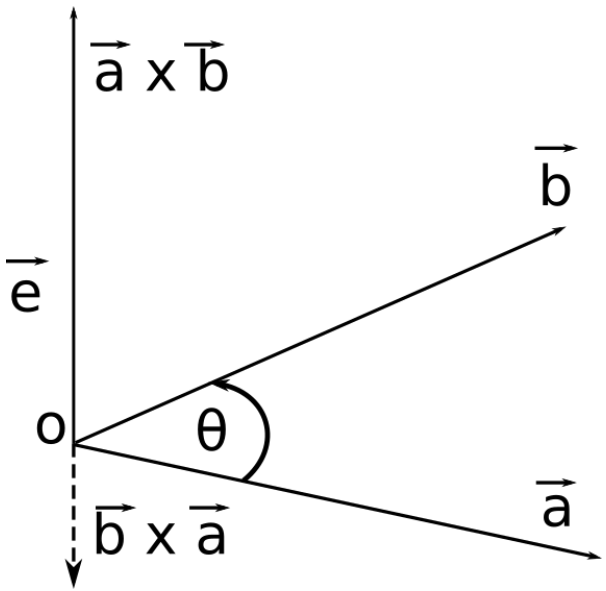


$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \Longrightarrow \boxed{\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3}$

$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ skalar

$$\cos\theta = \frac{\vec{a}\vec{b}}{ab} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}.\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Perkalian silang (Cross product)



Definisi:

$\vec{a} \times \vec{b} = (ab \sin\theta)\ \vec{e}; \quad 0 \leq \theta \leq \pi$

$\theta = \angle(\vec{a}, \vec{b})$ diukur dari $\vec{a}$ ke $\vec{b}$

$\vec{e}$ = vektor satuan $\perp$ bidangnya $\vec{a}$ dan $\vec{b}$.

$$\boxed{\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})}$$

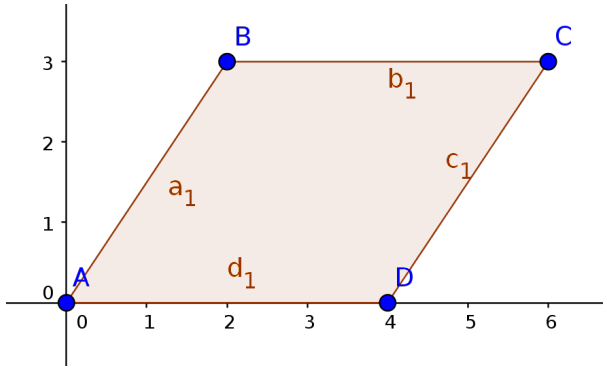
$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = \vec{k}; \quad \vec{j} \times \vec{k} = \vec{i}; \quad \vec{k} \times \vec{i} = \vec{j};$$

$$\vec{j} \times \vec{i} = -\vec{k}; \quad \vec{k} \times \vec{j} = -\vec{i}; \quad \vec{i} \times \vec{k} = -\vec{j};$$

Beberapa rumus [1]

- Luas jajaran genjang yang dibentuk $\vec{a}$ dan $\vec{b}$



$$L_{jajargenjang} = |\vec{a} \times \vec{b}| = ab \sin \theta$$

- Penulisan disingkat
- Rumus

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Beberapa rumus [2]

- Tiga vektor $\vec{a}$ $\vec{b}$ $\vec{c}$ membentuk paralelepipedum (balok miring)
- Rumus

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) - (\vec{a} \cdot \vec{b})\ \vec{c}}$$

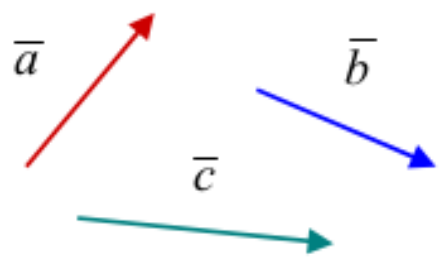
Contoh Soal

- Dapatkan sudut antara 2 vektor:
  $\vec{a} = 2\vec{i} + 2\vec{j} - \vec{k}$ *dan* $\vec{b} = 6\vec{i} - 3\vec{j} - 2\vec{k}$
- Jawab:

$$\vec{a} = \sqrt{(2)^2 + (2)^2 + (-1)^2} = 1;$$
$$\vec{b} = \sqrt{(6)^2 + (-3)^2 + (2)^2} = 1;$$
$$\vec{a} \cdot \vec{b} = (2)(6) + (2)(-3) + (-1)(2) = 4$$
$$\cos\ \theta = \frac{\vec{a}\vec{b}}{ab} = \frac{4}{(3)(7)} = 0.1905\ \Rightarrow\ \theta = 79^o$$

Soal

1. Diketahui vektor $\vec{a} = 3i + 4j$ dan vektor $\vec{b} = 2i + j$. Hitunglah harga-harga :
   - $\vec{a} + \vec{b}$
   - $\vec{b} + \vec{a}$
   - $\vec{a} - \vec{b}$
   - $\vec{b} - \vec{a}$
   - $|\vec{a}|$ dan $|\vec{b}|$
   - sudut $\vec{a}$
   - sudut $\vec{b}$
   - $\vec{a} \cdot \vec{b}$
   - $\vec{a} \times \vec{b}$
2. Diketahui vektor-vektor $\vec{a}, \vec{b}$ dan $\vec{c}$ seperti di bawah ini. Lukislah secara grafis operasi vektor :$\vec{a} - \vec{b} + 2.\vec{c}$ dan $3\vec{c} - 0,5(2\vec{a} - \vec{b})$.



Cartesian System

- a
- b